PROGRAMING THE TRAJECTORIES OF A SPACECRAFT

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The controlled motion of a spacecraft in a central gravitational field along a spatial trajectory with fixed ends is considered. The variable parameters of the trajectories of motion of the craft are approximated by polynomials in powers of the time whose coefficients are determined from the boundary conditions.

1. The motion of a spacecraft under the action of the controlling acceleration W applied to its center of mass O_1 is described by equations in a rotating right-handed orthogonal system $Oxyz$ whose y-axis coincides with the radius vector r constructed from the center of attraction O to the point O_1 and whose x-axis coincides with the direction of motion in such a way that the vector V of the absolute velocity of its center of mass lies in the plane xy. The orientation of the axes $Oxyz$ relative to the inertial axes $O\xi\eta\zeta$ is defined (Fig. 1) by the longitude Ω of the ascending node, the inclination i of the instantaneous orbital plane to the equator, and the range angle u. The equations of motion are

$$V_x^{\cdot} = W_x + \omega_z V_y, \qquad V_y^{\cdot} = W_y - \omega_z V_x - g$$
$$0 = W_z + \omega_y V_x, \quad \omega_z = -V_x / r, \quad g = g_0 (R_0 / r)^2 \tag{1.1}$$

The rates of change of the angles defining the orientation of the rotating axes relative to the inertial axes are defined by the differential equations

$$\frac{d\Omega}{dt} = \omega_y \frac{\sin u}{\sin i}, \quad \frac{di}{dt} = \omega_y \cos u, \quad \frac{du}{dt} = -\omega_z - \omega_y \sin u \operatorname{ctg} i \tag{1.2}$$

The present paper concerns a method of programing the spatial trajectory of motion of the spacecraft under given boundary conditions.

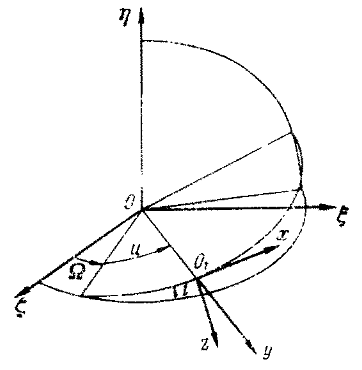

Fig. 1

The parameters of the trajectory of motion of the craft are expressed in the form of analytical relations realized by sufficiently simple control functions $W_x(t)$, $W_y(t)$, $W_z(t)$. These control functions represent the projections on the moving axes xyz of the controlling acceleration W applied to the center of mass of the craft.

The controlling acceleration is given by the expression

$$W = \sqrt{W_x^2 + W_y^2 + W_z^2} \tag{1.3}$$

The apparent velocity expended on control is given by

$$v(T) = \int_0^T W dt \tag{1.4}$$

where T is the duration of the controlled motion.

Fulfillment of the inequalities

$$W_{\min}(t) \leqslant W(t) \leqslant W_{\max}(t), \qquad v(T) \leqslant v_* \tag{1.5}$$

during motion over the control period $0 \leqslant t \leqslant T$ along the predicted craft trajectory are verified; here $W_{\min}(t)$ and $W_{\max}(t)$ are certain bounded functions of time. The control is considered permissible if these inequalities are fulfilled.

The problem of bringing the spacecraft to a specified point in the phase space within a given time has quite a simple solution if the parameters of motion of its center of mass over the time interval $0 \leqslant t \leqslant T$ are expressed in the form of the polynomials

$$V_x/r = a_0 + a_1 t + a_2 t^2, \qquad r = b_0 + b_1 t + b_2 t^2 + b_3 t^3 \tag{1.6}$$

These equations enable us to find the remaining parameters of the trajectory and to determine the coefficients $a_0,\ldots, a_2,\ldots, b_0,\ldots, b_3$ from the boundary conditions.

The polar angle in the plane of the involute between the initial and present positions of the radius vector of the center of mass of the craft is given by the equation

$$J = -\int_0^t \omega_z d\tau \tag{1.7}$$

This equation is integrated with allowance for the fourth equation of (1.1) and the first equation of (1.6).

Carrying out this integration, we obtain

$$J = t(a_0 + {}^1/_2 a_1 t + {}^1/_3 a_2 t^2) \tag{1.8}$$

The coefficients of Eq. (1.8) and of the first equation of (1.6) are determined from the boundary conditions for $t = 0$ and $t = T$,

$$a_0 = \frac{V_{x0}}{r_0}, \quad a_1 = \frac{2}{T}\left(-\frac{V_{xk}}{r_k} - 2a_0 + \frac{3}{T} J_k\right)$$
$$a_2 = \frac{3}{T^2}\left(\frac{V_{xk}}{r_k} + a_0 - \frac{2}{T} J_k\right) \tag{1.9}$$

We assume that the quantities V_{x0}, V_{xk}, r_0, r_k, $r_0^{\cdot}$, $r_k^{\cdot}$, J_k and T at the beginning of controlled motion are known.

Differentiating the second equation of (1.6), we obtain the vertical velocity of the craft and the relative acceleration along the radius vector of its center of mass,

$$r^{\cdot} = V_y^{\cdot} = b_1 + 2b_2 t + 3b_3 t^2, \qquad V_y^{\cdot} = 2b_2 + 6b_3 t \tag{1.10}$$

We determine the coefficients $b_0,\ldots, b_3$ by solving these equations simultaneously with the second equation of (1.6) for the prescribed boundary conditions for $t = 0$ and $t = T$,

$$b_0 = r_0, \quad b_1 = r_0^{\cdot}, \quad b_2 = \frac{1}{T}\left(\frac{3h}{T} - r_k^{\cdot} - 2r_0^{\cdot}\right)$$
$$b_3 = \frac{1}{T^2}\left(r_k^{\cdot} + r_0^{\cdot} - \frac{2h}{T}\right), \qquad h = r_k - r_0 \tag{1.11}$$

The projection of the absolute velocity vector $\mathbf{V}$ on the direction of motion is defined as the product of the first and second equations of (1.6),

$$V_x = c_0 + c_1 t + \ldots + c_5 t^5 \tag{1.12}$$

Computing the coefficients $c_0,\ldots, c_5$, we obtain

$$c_0 = a_0 b_0, \qquad c_1 = a_0 b_1 + a_1 b_0, \quad c_2 = a_0 b_2 + a_1 b_1 + a_2 b_0$$
$$c_3 = a_0 b_3 + a_1 b_2 + a_2 b_1, \quad c_4 = a_1 b_3 + a_2 b_2, \quad c_5 = a_2 b_3 \tag{1.13}$$

The relative acceleration of the craft in the direction of motion is found by differentiating Eq. (1.12),

$$V_x^{\cdot} = c_1 + 2c_2 t + \ldots + 5c_5 t^4 \tag{1.14}$$

Multiplication of the first equation of (1.6) by the first equation of (1.10) gives us the rotational acceleration of the craft,

$$V_x V_y / r = p_0 + p_1 t + \ldots + p_4 t^4 \tag{1.15}$$

Computing the coefficients $p_0, \ldots, p_4$, we obtain

$$p_0 = a_0 b_1, \quad p_1 = 2a_0 b_2 + a_1 b_1, \quad p_2 = 3a_0 b_3 + 2a_1 b_2 + a_2 b_1 \tag{1.16}$$
$$p_3 = 3a_1 b_3 + 2a_2 b_2, \quad p_4 = 3a_2 b_3$$

Multiplying the first equation of (1.6) by Eq. (1.12), we obtain the centripetal acceleration along the trajectory of motion of the craft,

$$V_x^2 / r = q_0 + q_1 t + \ldots + q_7 t^7 \tag{1.17}$$

Computation of the coefficients $q_0, \ldots, q_7$ with allowance for (1.13) yields

$$q_0 = a_0^2 b_0, \quad q_1 = a_0(2a_1 b_0 + a_0 b_1), \qquad q_2 = 2a_0(a_1 b_1 + a_2 b_0) + a_0^2 b_2 + a_1^2 b_0$$
$$q_3 = 2(a_1 a_2 b_0 + a_0 a_2 b_1 + a_0 a_1 b_2) + a_0^2 b_3 + a_1^2 b_1$$
$$q_4 = 2(a_1 a_2 b_1 + a_0 a_2 b_2 + a_0 a_1 b_3) + a_1^2 b_2 + a_2^2 b_0 \tag{1.18}$$
$$q_5 = 2a_2(a_1 b_2 + a_0 b_3) + a_1^2 b_3 + a_2^2 b_1, \quad q_6 = a_2(a_2 b_2 + 2a_1 b_3), \quad q_7 = a_2^2 b_3$$

The control function $W_x(t)$ is found from the first equation of (1.1) with allowance for Eqs. (1.14) and (1.15),

$$W_x = c_1 + p_0 + (2c_2 + p_1)t + \ldots + (5c_5 + p_4)t^4 \tag{1.19}$$

The control function $W_y(t)$ is found from the second equation of (1.1) with allowance for the second equation of (1.10) and Eq. (1.17),

$$W_y = 2b_2 - q_0 + (6b_3 - q_1)t - q_2 t^2 - \ldots - q_7 t^7 + g \tag{1.20}$$

The gravitational acceleration g in this equation is a known function of time by virtue of the fifth equation of (1.1) and the second equation of (1.6).

Control laws (1.19), (1.20) completely define the motion of the craft in the plane of the involute along the chosen trajectory with fixed ends.

2. The control law for the motion of the orbital plane of the craft (according to [1]) is

$$W_z = K V_x^2 / r \tag{2.1}$$

With this control law Eqs. (1.2) are integrable independently of Eqs. (1.1), which makes it possible to program the spatial trajectory of the spacecraft.

Programing of the controlled motion of the orbital plane begins with prescribing the shape of the craft trajectory on the surface of a unit sphere.

The locus of the representing point over the segment of the trajectory where $W_z = 0$ lies along a great-circle arc, and the angle J is determined from the boundary conditions.

The representing point is defined as the point of intersection of the radius vector of the center of mass of the craft with the surface of the unit sphere.

For $W_z \neq 0$ the character of the locus of the representing point is determined from the formulas derived in [1].

$$J = \int_{x_0}^{x_k} \frac{dx \operatorname{sign}(K \cos u)}{\sqrt{-(1+K^2)x^2 + 2kx + K^2 - k^2}} \qquad (x = \cos i)$$

$$\cos i - K \sin u \sin i = k, \qquad k = \cos i_0 - K \sin u_0 \sin i_0 \tag{2.2}$$

$$\Omega_k - \Omega_0 = \int_{x_0}^{x_k} \frac{(x-k) \operatorname{sign}(K \cos u)\, dx}{(x^2 - 1)\sqrt{-(1+K^2)x^2 + 2kx + K^2 - k^2}}$$

To simplify the subsequent operations involved in determining the shape of the trajectory described by the representing point we orient the inertial axes in such a way that

$$u_0 = \Omega_0 = i_0 = 0$$

at the initial instant of orbital-plane motion control.

The third relation of (2.2) then gives us the constant $k = 1$.

Let us assume that the extremal value i_* of the angle of inclination of the orbital plane is not attained during the control period. Then, integrating the first and last equations of (2.2), we obtain

$$J = \frac{1}{\sqrt{1+K^2}} \arccos\left[\frac{(1+K^2)\cos i - 1}{K^2}\right]$$
$$\Omega = \frac{1}{2} \arccos\left[\frac{1}{K^2}\left(-\frac{4}{\cos i + 1} + 2 + K^2\right)\right] \qquad (2.3)$$

Transforming the second equation of (2.2) for $k = 1$, we obtain the range angle,

$$u = \frac{1}{2} \arccos\left[\frac{1}{K^2}\left(-\frac{4}{\cos i + 1} + 2 + K^2\right)\right] \qquad (2.4)$$

Combining the resulting equation with the second equation of (2.3), we obtain the identity

$$\Omega = u \qquad (2.5)$$

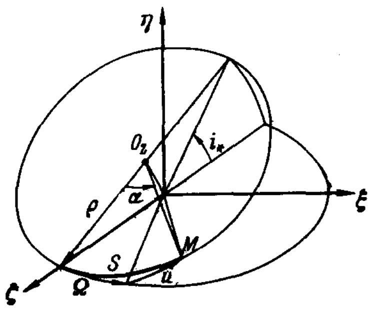

Fig. 2

It is evident from geometric considerations that this identity can be fulfilled at any instant only if the instantaneous orbital plane is in continuous contact with the surface of a circular cone with its vertex at the center of attraction. The intersection of the surface of a circular cone with the surface of a unit sphere is a small circle.

Let us demonstrate the validity of this geometric argument.

Figure 2 shows a circular cone with its vertex at the origin of the inertial coordinate system ξ, η, ζ.

During controlled motion the representing point M describes the arc S coinciding with the base of the circular cone.

We see from the geometric construction that the extremal inclination i_* of the orbital plane corresponds to $\Omega = u = \pi/2$.

The same conclusion is obtained by considering the second equation of (1.2) and Eq. (2.5). The base angle of the circular cone is $i_*/2$.

The radius of the small circle which is the base of the circular cone is

$$\rho = 1 \cdot \cos\ (i_*/2) \qquad (2.6)$$

From the second equation of (2.2) for $i = i_*$ and $u = \pi/2$ we find that

$$\cos(i_*/2) = 1 / \sqrt{1+K^2} \qquad (2.7)$$

Comparing this expression with formula (2.6), we obtain the equation

$$\rho = 1 / \sqrt{1+K^2} \qquad (2.8)$$

The length of the small-circle arc on the control segment (Fig. 2) is

$$S = \rho\alpha \qquad (2.9)$$

On the other hand we have the equation

$$S = 1 \cdot J$$

Transforming this formula with allowance for (2.8) and (2.9) we obtain

$$\alpha = J\sqrt{1+K^2} \tag{2.10}$$

The construction of Fig. 2 with allowance for Eq. (2.7) yield the relation

$$\alpha = \arccos\left[\frac{(1+K^2)\cos i - 1)}{K^2}\right] \tag{2.11}$$

A similar relation can be obtained from the first equation of (2.3) with allowance for (2.10).

We are now fully convinced of the validity of the geometric interpretation of the character of motion of the orbital plane of the craft.

Let us consider the basic case of transfer of the craft from the initial orbital plane to the prescribed plane where the control function W_z is of constant sign and where the motion occurs over a unique small-circle arc.

The craft can be transferred from the initial orbital plane to the prescribed one along an infinite number of arcs of differing curvature. This makes it necessary to optimize the trajectory of motion of the orbital plane.

As our optimality criterion we take the functional

$$v_z = \int_{t_1}^{t_2} |W_z|\, dt = \min$$

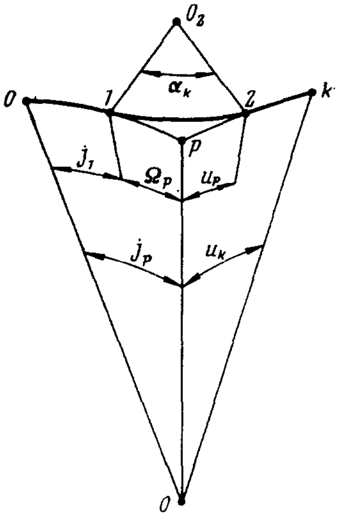

Fig. 3

Making use of (2.1), we can rewrite this functional as

$$v_z = \int_{t_1}^{t_2} |K| \frac{V_x^2}{r}\, dt = \min \tag{2.12}$$

The unknowns in this equation are the coefficient K and the integration limits t_1 and t_2.

The initial and final instants t_1 and t_2 of controlled motion of the orbital plane are functions of the coefficient K.

The dependence of the integration limits t_1 and t_2 on the parameter K can be found by means of a geometric construction (Fig. 3). The motion over the segments $0-1$ and $2-k$ in the figure is along great-circle arcs; the motion over the segment $1-2$ is over a small-circle arc.

The quantity J_p is the angular distance along a great-circle arc between the initial position of the radius vector of the center of mass of the craft and the line of intersection of the initial and prescribed orbital planes.

By virtue of (2.5) and (2.3)

$$\Omega_p = u_p = \frac{1}{2}\arccos\left[\frac{1}{K^2}\left(-\frac{4}{\cos i_k + 1} + 2 + K^2\right)\right]$$

We infer from this expression, Eq. (1.8), the first equation of (2.3), and the geometric constructions of Fig. 3 that

$$J_1 = J_p - \frac{1}{2}\arccos\left[\frac{1}{K^2}\left(-\frac{4}{\cos i_k + 1} + 2 + K^2\right)\right] = t_1\left(a_0 + \frac{1}{2}a_1t_1 + \frac{1}{3}a_2t_1^2\right)$$

$$J_2 = J_1 + \frac{1}{\sqrt{1+K^2}}\arccos\left[\frac{(1+K^2)\cos i_k - 1}{K^2}\right] = t_2\left(a_0 + \frac{1}{2}a_1t_2 + \frac{1}{3}a_2t_2^2\right) \quad (2.13)$$

Differentiating functional (2.12) with respect to the parameter K and equating the result to zero, we obtain the following equation for determining the optimal value of the quantity K :

$$\frac{dv_z}{dK} = \operatorname{sign}(K)\int_{t_1}^{t_2}\frac{V_x^2}{r}\,dt + |K|\left(\frac{dt_2}{dK}\frac{V_{x2}^2}{r_2} - \frac{dt_1}{dK}\frac{V_{x1}^2}{r_1}\right) = 0 \quad (2.14)$$

Differentiation of formulas (2.13) with respect to the parameter K, we obtain the derivatives occurring in Eq. (2.14),

$$\frac{dt_1}{dK} = \frac{r_1}{V_{x1}}\frac{a}{K\sqrt{K^2-a^2}} \qquad \left(a = \sqrt{\frac{1-\cos i_k}{1+\cos i_k}}\right) \quad (2.15)$$

$$\frac{dt_2}{dK} = \frac{r_2}{V_{x2}}\left\{\frac{a}{K\sqrt{K^2-a^2}}\frac{K^2-1}{1+K^2} - \frac{K}{(1+K^2)^{3/2}}\arccos\left[\frac{(1+K^2)\cos i_k - 1}{K^2}\right]\right\}$$

The above expressions enable us to transform formula (2.14) into

$$\frac{dv_z}{dK} = \int_{t_1}^{t_2}\frac{V_x^2}{r}dt + V_{x2}\left\{\frac{a}{\sqrt{K^2-a^2}}\frac{K^2-1}{1+K^2} - \right. \quad (2.16)$$

$$\left. - \frac{K^2}{(1+K^2)^{3/2}}\arccos\left[\frac{(1+K^2)\cos i_k - 1}{K^2}\right]\right\} - V_{x1}\frac{a}{\sqrt{K^2-a^2}} = 0$$

This equation has a solution as $K \to \infty$, because each of its terms tends to zero. In this case Eqs. (2.13) yield $J_1 = J_2 = J_p, \quad t_1 = t_2 = t_p$

Here t_p is the time required for the craft to reach the line of intersection of the initial and prescribed orbital planes in moving along the great-circle arc $0p$.

Transforming integral (2.12) with allowance for (2.10), (1.7), and the fourth equation of (1.1), we obtain

$$v_z = \frac{|K|}{\sqrt{1+K^2}}\int_0^{\alpha_k} V_x\,d\alpha \quad (2.17)$$

With allowance for (2.11) as $K \to \infty$, (2.17) becomes

$$v_z = V_{xp}\, i_k$$

In fact, however, K is a finite quantity by virtue of the first inequality of (1.5).

The quantity K must be maximized in order to minimize the expenditure on control of the apparent velocity v_z. The coefficient K is chosen by the method of successive approximations with allowance for inequalities (1.5). The next step is to use Eqs. (2.13) to find the initial and final instants t_1 and t_2 of controlled motion of the orbital plane of the craft.

3. An approximate analytical evaluation of the permissibility of the control on the basis of inequalities (1.5) is carried out by expressing the function $W(t)$ as a polynomial of degree n. This entails integrating Eq. (1.4) and finding a formula for determining the apparent velocity expended on control, i.e. $v(T)$.

The polynomial describing the function $W(t)$ is obtained by expanding the right side of Eq. (1.3) in a Taylor series and discarding the small terms.

To simplify the formulas defining the coefficients of the series we expand the gravitational acceleration in (1.20) in a Taylor series at the point r_0,

$$g = g^\circ \sum_{n=0}^{\infty} S_n t^n, \qquad g^\circ = g_0 (R_0 / r_0)^2 \tag{3.1}$$

This series converges very rapidly for real controlled-motion trajectories, since the relative change in the quantity r over the control segment is insignificant.

Series (3.1) is approximated with sufficient accuracy by the polynomial

$$g = g^\circ(s_0 + s_1 t + \ldots + s_3 t^3) \tag{3.2}$$

The coefficients $s_0, \ldots, s_3$ of this polynomial computed with allowance for Eqs.(1.10) are given by the expressions

$$s_0 = 1, \quad s_1 = -2b_1/b_0, \; s_2 = (3b_1^2 - 2b_2 b_0) / b_0^2 \tag{3.3}$$
$$s_3 = 2(-2b_1^3 + 3b_0 b_1 b_2 - b_0^2 b_3) / b_0^3$$

By virtue of the smallness of the relative change in the radius r and the boundedness of the function $W(t)$ we can assume that the coefficients q_5, q_6, q_7 in (1.17) are equal to zero.

We can now write Eqs. (1.19), (1.20) and (2.1) with allowance for (3.2) and (1.17) in the form

$$W_x = \alpha_0 + \alpha_1 t + \ldots + \alpha_4 t^4, \quad W_y = \beta_0 + \beta_1 t + \ldots + \beta_4 t^4 \tag{3.4}$$
$$W_z = \gamma_0 + \gamma_1 t + \ldots + \gamma_4 t^4$$

The coefficients of these polynomials are given by the formulas

$$\alpha_n = (n+1)\, c_{n+1} + p_n \quad (n = 0, 1, 2, 3, 4)$$
$$\beta_n = (n+1)(n+2)\, b_{n+2} - q_n + g^0 s_n, \qquad \gamma_n = K q_n \tag{3.5}$$

The coefficients b_4, b_5, b_6, s_4 are equal to zero.

We can now find the function $W(t)$ by substituting Eqs. (3.4) into (1.3),

$$W = (j_0 + j_1 t + \ldots + j_8 t^8)^{1/2} \tag{3.6}$$

The coefficients $j_0, \ldots, j_8$ turn out to be

$$j_0 = \alpha_0^2 + \beta_0^2 + \gamma_0^2, \quad j_1 = 2(\alpha_0\alpha_1 + \beta_0\beta_1 + \gamma_0\gamma_1)$$
$$j_2 = 2(\alpha_0\alpha_2 + \beta_0\beta_2 + \gamma_0\gamma_2) + \alpha_1^2 + \beta_1^2 + \gamma_1^2$$
$$j_3 = 2(\alpha_0\alpha_3 + \beta_0\beta_3 + \gamma_0\gamma_3 + \alpha_1\alpha_2 + \beta_1\beta_2 + \gamma_1\gamma_2) \tag{3.7}$$
$$j_4 = 2(\alpha_0\alpha_4 + \beta_0\beta_4 + \gamma_0\gamma_4 + \alpha_1\alpha_3 + \beta_1\beta_3 + \gamma_1\gamma_3) + \alpha_2^2 + \beta_2^2 + \gamma_2^2$$
$$j_5 = 2(\alpha_1\alpha_4 + \beta_1\beta_4 + \gamma_1\gamma_4 + \alpha_2\alpha_3 + \beta_2\beta_3 + \gamma_2\gamma_3)$$
$$j_6 = 2(\alpha_2\alpha_4 + \beta_2\beta_4 + \gamma_2\gamma_4) + \alpha_3^2 + \beta_3^2 + \gamma_3^2$$
$$j_7 = 2(\alpha_3\alpha_4 + \beta_3\beta_4 + \gamma_3\gamma_4), \quad j_8 = \alpha_4^2 + \beta_4^2 + \gamma_4^2$$

The function $W(t)$ has first-order discontinuities at the initial and final instants t_1 and t_2 of controlled motion of the orbital plane of the craft. This means that its Taylor-series expansion must be taken separately over each continuous segment, and that the integration limits in formula (1.4) are split into $0 < t_1 < t_2 < T$. To simplify our expressions we assume that $t_1 = 0$, $t_2 = T$. Next, expanding the right side of Eq. (3.6) in a Taylor series at the point t_0 (where $W = W_{\max}$), we obtain

$$W(t) = W(t_0) \sum_{n=0}^{\infty} l_n \Delta t^n \qquad (\Delta t = t - t_0) \tag{3.8}$$

This series converges rapidly for permissible trajectories by virtue of the boundedness of $W(t)$.

Practically, with many real trajectories it is sufficient to retain three terms only,

$$W = W(t_0)(l_0 + l_1 \Delta t + l_2 \Delta t^2), \qquad W(t_0) = (j_0 + j_1 t_0 + \ldots + j_8 t_0^8)^{1/2} \tag{3.9}$$

Computing the coefficients l_0, l_1, l_2 of this polynomial, we obtain

$$l_0 = 1, \; l_1 = \frac{1}{2W^2(t_0)} \sum_{n=0}^{7} (n+1) j_{n+1} t_0^n$$

$$l_2 = \frac{1}{4W^2(t_0)} \sum_{n=0}^{6} (n+1)(n+2) j_{n+2} t_0^n - \frac{1}{2} l_1^2$$

Integrating Eq. (1.4) with allowance for (3.9), we obtain the apparent velocity expended on control,

$$v(T) = TW(t_0)[l_0 + l_1(^1/_2 T - t_0) + l_2(^1/_3 T^2 - t_0 T + t^2_0)] \tag{3.10}$$

The permissibility of a chosen trajectory of craft motion with fixed ends can be verified by substituting (3.6), (3.10) into inequalities (1.5).

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ON THE AUTOOSCILLATIONS OF GYROSCOPIC STABILIZERS

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The system of third-order differential equations describing the motion of a single-axis gyro stabilizer with a floating integrating gyro is investigated. The stabilizer motor is controlled by means of a contact device with a dead zone δ. It is shown that for a sufficiently small δ the system has a closed trajectory corresponding to the autooscillations of the gyro stabilizer. The domain of immersion of the closed trajectory in the phase space is specified.

The autooscillations of gyro stabilizers were investigated in [1–4]. The author of [1, 2] analyzed the motion of a gyro stabilizer in the case of a relay-type stabilizer motor control. He determined the parameters and investigated the stability of the periodic motion by the method of point transformations. The author of [3, 4] treated the problem by the harmonic linearization method of E. P. Popov in conjunction with electronic modelling. The primary emphasis in these studies was on computing the periodic motion. In the theory of gyroscopic instruments employing autooscillatory operating modes it is especially important to investigate the conditions of existence of closed trajectories of the differential equations of gyro system motion, to prove the existence of